



I. (20 Points) Given $A = \begin{bmatrix} 0 & 0 & -2 & 0 & 7 & 12 \\ 2 & 4 & -10 & 6 & 12 & 28 \\ 2 & 4 & -5 & 6 & -5 & -1 \end{bmatrix}$

- Find the rank and nullity of A.
- Find the basis for the row space, column space and nullspace of A.

II. (20 Points) Let $A = \begin{bmatrix} 1 & -2 & 3 \\ 2 & -3 & 2 \\ -1 & 1 & 2 \end{bmatrix}$

- Determine A^{-1} using the cofactor method.
- Use the result of part A to solve the following system :

$$xy - 2\sqrt{y} + 3zy = 8$$

$$2xy - 3\sqrt{y} + 2zy = 7$$

$$-xy + \sqrt{y} + 2zy = 4$$

III. (10 Points) Let V be the vector space of polynomials of degree 3. Determine whether the polynomials $u = t^3 - 3t^2 + 5t + 1$, $v = t^3 - t^2 + 8t + 2$, and $w = 2t^3 + 4t^2 + 9t + 5$ are dependent or independent?

IV. (10 Points) Find a subset of the vectors that forms a basis for the space spanned by the vectors:

$$\vec{v}_1 = (1; 0; 1; 1) \quad \vec{v}_2 = (-3; 3; 7; 1) \quad \vec{v}_3 = (-1; 3; 9; 3) \quad \text{and} \quad \vec{v}_4 = (-5; 3; 5; -1),$$

Express each vector that is not in the basis as a linear combination of the basis vectors.

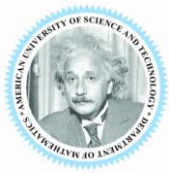
V. (10 Points) Evaluate the determinant of $A = \begin{bmatrix} 2 & 5 & -3 & -2 \\ -2 & -3 & 2 & -5 \\ 1 & 3 & -2 & 2 \\ -1 & -6 & 4 & 3 \end{bmatrix}$

VI. (10 Points) Find the coordinate vector of A relative to the basis $S = \{A_1, A_2, A_3, A_4\}$ where:

$$A = \begin{bmatrix} 2 & 0 \\ -1 & 3 \end{bmatrix}; \quad A_1 = \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix}; \quad A_2 = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}; \quad A_3 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}; \quad A_4 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

VII. (10 Points) Determine the dimension of and a basis for the solution space W of the

homogenous system:
$$\begin{bmatrix} 1 & 1 & 4 & 1 & 2 \\ 0 & 1 & 2 & 1 & 1 \\ 0 & 0 & 0 & 1 & 2 \\ 1 & -1 & 0 & 0 & 2 \\ 2 & 1 & 6 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ u \\ v \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$



VIII. (10 Points) Show the following:

- a) The homogeneous system $\begin{cases} (a-r)x + dy = 0 \\ cx + (b-r)y = 0 \end{cases}$ has a nontrivial solution if and only if r satisfies the equation: $(a-r)(b-r) - cd = 0$
- b) $\det \begin{bmatrix} a+b & ab & 0 \\ 1 & a+b & ab \\ 0 & 1 & a+b \end{bmatrix} = \frac{a^4 - b^4}{a-b}$