I. (20 Points) Given $A=\begin{bmatrix}0 & 0 & -2 & 0 & 7 & 12\\ 2 & 4 & -10 & 6 & 12 & 28\\ 2 & 4 & -5 & 6 & -5 & -1\end{bmatrix}$

a) Find the rank and nullity of A.
b) Find the basis for the row space, column space and nullspace of A.

II. (20 Points) Let $A=\begin{bmatrix}1 & -2 & 3\\ 2 & -3 & 2\\ -1 & 1 & 2\end{bmatrix}$

a) Determine A^{-1} using the cofactor method.
b) Use the result of part A to solve the following system :

$$\begin{aligned} xy-2\sqrt{y}+3zy&=8\\ 2xy-3\sqrt{y}+2zy&=7\\ -xy+\sqrt{y}+2zy&=4 \end{aligned}$$

III. (10 Points) Let V be the vector space of polynomials of degree 3. Determine whether the polynomials $u=t^3-3t^2+5t+1$, $v=t^3-t^2+8t+2$, and $w=2t^3+4t^2+9t+5$ are dependent or independent?

IV. (10 Points) Find a subset of the vectors that forms a basis for the space spanned by the vectors:

$\vec{v_1}=(1;0;1;1)$ $\vec{v_2}=(-3;3;7;1)$ $\vec{v_3}=(-1;3;9;3)$ *and* $\vec{v_4}=(-5;3;5;-1)$,

Express each vector that is not in the basis as a linear combination of the basis vectors.

V. (10 Points) Evaluate the determinant of $A=\begin{bmatrix}2 & 5 & -3 & -2\\ -2 & -3 & 2 & -5\\ 1 & 3 & -2 & 2\\ -1 & -6 & 4 & 3\end{bmatrix}$

VI. (10 Points) Find the coordinate vector of A relative to the basis $S=\{A_1,A_2,A_3,A_4\}$ where:

$$A=\begin{bmatrix}2 & 0\\ -1 & 3\end{bmatrix};\quad A_1=\begin{bmatrix}-1 & 1\\ 0 & 0\end{bmatrix};\quad A_2=\begin{bmatrix}1 & 1\\ 0 & 0\end{bmatrix};\quad A_3=\begin{bmatrix}0 & 0\\ 1 & 0\end{bmatrix};\quad A_4=\begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix}$$

VII. (10 Points) Determine the dimension of and a basis for the solution space W of the homogenous system: $\begin{bmatrix}1 & 1 & 4 & 1 & 2\\ 0 & 1 & 2 & 1 & 1\\ 0 & 0 & 0 & 1 & 2\\ 1 & -1 & 0 & 0 & 2\\ 2 & 1 & 6 & 0 & 1\end{bmatrix}\begin{bmatrix}x\\ y\\ z\\ u\\ v\end{bmatrix}=\begin{bmatrix}0\\ 0\\ 0\\ 0\\ 0\end{bmatrix}$

VIII. (10 Points) Show the following:

a) The homogeneous system $\begin{cases}(a-r)x+dy=0\\ cx+(b-r)y=0\end{cases}$ has a nontrivial solution if and only if r satisfies the equation: $(a-r)(b-r)-cd=0$

b) $\det\begin{bmatrix} a+b & ab & 0 \\ 1 & a+b & ab \\ 0 & 1 & a+b \end{bmatrix} = \dfrac{a^4-b^4}{a-b}$